



Quadratic Equations – Factorisation – With Coefficients – Demonstration

This resource provides animated demonstrations of the mathematical method.

We factorise a quadratic expression to help us solve a quadratic equation.

$$2x^2 + 7x + 3 = 0 \quad \Rightarrow \quad (2x + 1)(x + 3) = 0$$

What terms are in each bracket?

How many possibilities are there?

$$\text{Prime } 3x^2 + 7x + 2 = 0 \text{ Prime}$$

↑

$$(3x \quad)(x \quad) = 0 \quad \checkmark$$

$$(3x + 2)(x + 1) = 0$$

**Must** have  $3x$  &  $x$  in either bracket.

**Must** have  $2$  &  $1$  in either bracket.

They must either be both positive or both negative.

**Must** be both positive.

What terms are in each bracket?

How many possibilities are there?

$$\text{Prime } 5x^2 + 8x \text{ Prime } + 3 = 0$$

↑

$$(5x \quad)(x \quad) = 0$$

$$(5x + 3)(x + 1) = 0$$



**Must** have  $5x$  &  $x$  in either bracket.

**Must** have 3 & 1 in either bracket.

They must either be both positive or both negative.

**Must** be both positive.

What terms are in each bracket?

How many possibilities are there?

$$\text{Prime } 2x^2 - 7x \text{ Prime } + 5 = 0$$

↑

$$(2x \quad)(x \quad) = 0$$

$$(2x - 5)(x - 1) = 0$$



**Must** have  $2x$  &  $x$  in either bracket.

**Must** have  $5$  &  $1$  in either bracket.

They must either be both positive or both negative.

**Must** be both negative.

What terms are in each bracket?

How many possibilities are there?

$$\textcircled{3}x^2 + 8x\textcircled{+ 4} = 0$$

Prime



$$(3x \quad )(x \quad ) = 0$$

$$(3x + 4)(x + 1) = 0$$

$$(3x + 2)(x + 2) = 0$$

$$(3x + 2)(x + 2) = 0$$



**Must** have  $3x$  &  $x$  in either bracket.

**Must** have 4 & 1, or 2 & 2 in either bracket.

They must either be both positive or both negative.

**Must** be both positive.

What terms are in each bracket?

How many possibilities are there?

$$5x^2 + 11x + 6 = 0$$

Prime



$$(5x \quad)(x \quad) = 0$$

$$(5x + 6)(x + 1) = 0$$



$$(5x + 2)(x + 3) = 0$$

$$(5x + 3)(x + 2) = 0$$

**Must** have  $5x$  &  $x$  in either bracket.

**Must** have 6 & 1, or 2 & 3 in either bracket.

They must either be both positive or both negative.

**Must** be both positive.

What terms are in each bracket?

How many possibilities are there?

$$\text{Prime } 3x^2 + 2x \text{ Prime } - 5 = 0$$

↑

$$(3x \quad)(x \quad) = 0$$

$$(3x + 5)(x - 1) = 0$$




**Must** have  $3x$  &  $x$  in either bracket.

**Must** have  $5$  &  $1$  in either bracket.

They may be either positive or negative.

What terms are in each bracket?

How many possibilities are there?

$$4x^2 + 20x + 9 = 0$$


$$(4x \quad)(x \quad) = 0$$

$$(4x + 3)(x + 3) = 0$$

$$(2x \quad)(2x \quad) = 0 \quad \checkmark$$

$$(2x + 3)(2x + 3) = 0$$

**Must** have  $4x$  &  $x$ , or  $2x$  &  $2x$  in either bracket.

**Must** have  $9$  &  $1$ , or  $3$  &  $3$  in either bracket.

They must either be both positive or both negative.

**Must** be both positive.



For each equation, write down all the double bracket possibilities.  
Expand the double brackets to find the correct choice.

$$2x^2 + 5x + 3 = 0$$

$$(2x + 3)(x + 1) = 0 \checkmark$$

$$(2x + 1)(x + 3) = 0$$

$$3x^2 - 22x + 7 = 0$$

$$(3x - 7)(x - 1) = 0$$

$$(3x - 1)(x - 7) = 0 \checkmark$$

$$5x^2 + 12x + 4 = 0$$

$$(5x + 4)(x + 1) = 0$$

$$(5x + 1)(x + 4) = 0$$

$$(5x + 2)(x + 2) = 0 \checkmark$$

$$(5x + 2)(x + 2) = 0$$

For each equation, write down all the double bracket possibilities.  
Expand the double brackets to find the correct choice.

$$3x^2 + 11x + 6 = 0$$

$$(3x + 6)(x + 1) = 0$$

$$(3x + 1)(x + 6) = 0$$

$$(3x + 2)(x + 3) = 0$$

$$(3x + 3)(x + 2) = 0$$

$$6x^2 + 43x + 7 = 0$$

$$(6x + 7)(x + 1) = 0$$

$$(6x + 1)(x + 7) = 0$$

$$(3x + 7)(2x + 1) = 0$$

$$(2x + 1)(3x + 7) = 0$$

$$4x^2 - 9x + 4 = 0$$

$$(4x - 4)(x - 1) = 0$$

$$(4x - 1)(x - 4) = 0$$

$$(4x - 2)(x - 2) = 0$$

$$(2x - 4)(2x - 1) = 0$$

$$(2x - 1)(2x - 4) = 0$$

$$(2x - 2)(2x - 2) = 0$$

$$6x^2 - 2x - 8 = 0$$

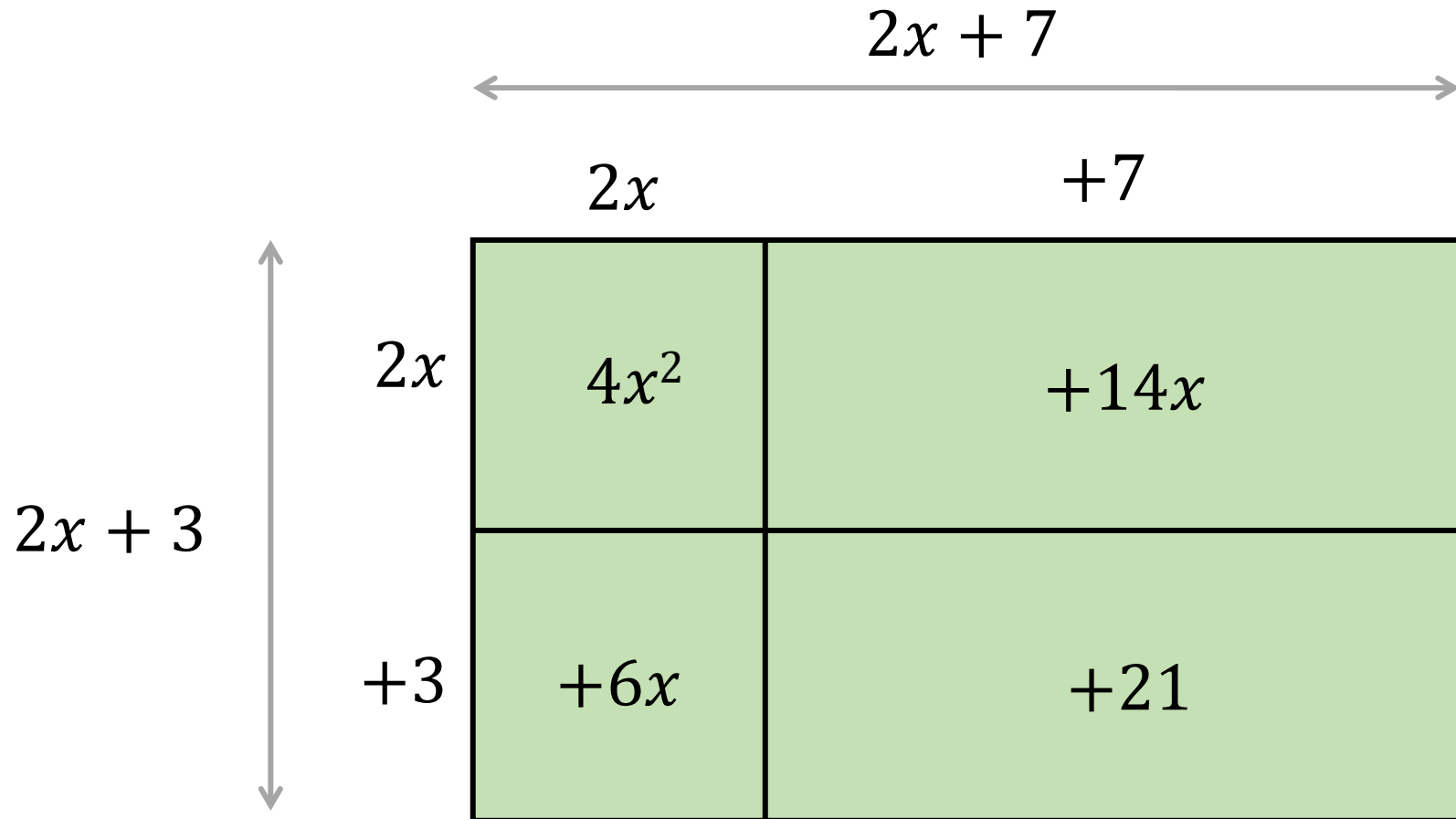
$$(6x + 4)(x - 2) \quad (2x + 4)(3x - 2)$$

$$(6x + 2)(x - 4) \quad (2x + 2)(3x - 4)$$

$$(6x + 8)(x - 1) \quad (2x + 8)(3x - 1)$$

$$(6x + 1)(x - 8) \quad (2x + 1)(3x - 8)$$

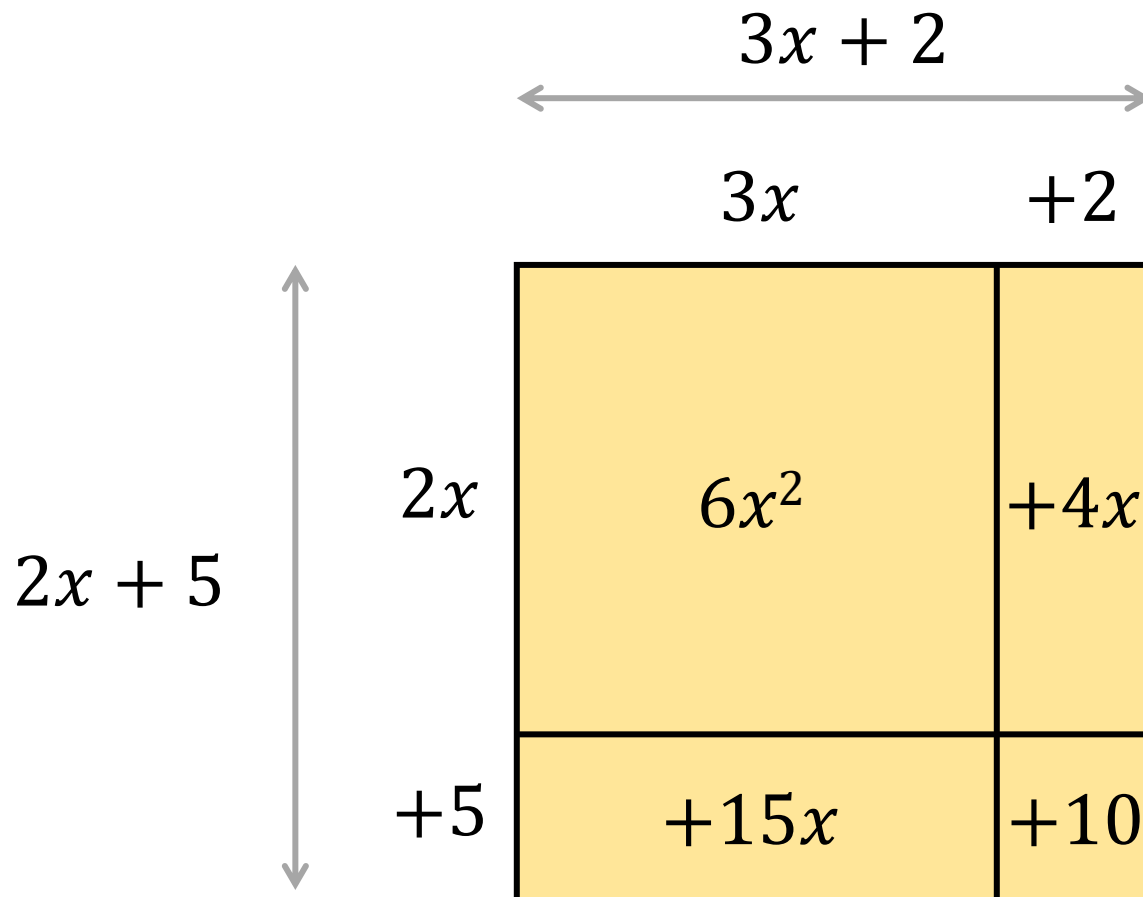
Find an expression for the area of this rectangle.



$$\text{Area} = 4x^2 + 20x + 21$$

$$= (2x + 3)(2x + 7)$$

Find expressions for the length & width of this rectangle.



$$\begin{aligned}\text{Area} &= 6x^2 + 19x + 10 \\ &= (3x + 2)(2x + 5)\end{aligned}$$

Completing an algebraic multiplication table.

	$2x$	$-5$
$x$		
$+4$		

	$3x$	$-2$
$4x$		
$-3$		

In **reverse**, how can we complete these tables with **factors**?

	HCF	HCF
HCF	$2x^2$	$+3x$
HCF	$+10x$	$+15$

	$3x^2$	$+12x$
	$-2x$	$-8$

How can we check the positive & negative signs are correct?

Complete each table with the **highest common factors** of each column & row.  
 How can we **check** we have positive & negative values correct?

a)

	$x$	
	$x^2$	$+2x$
$+4$	$+4x$	$+8$

b)

	$x^2$	$-2x$
	$+4x$	$-8$

c)

	$2x^2$	$+x$
	$-8x$	$-4$

d)

	$2x^2$	$-6x$
	$-3x$	$+9$

e)

	$4x^2$	$+10x$
	$+2x$	$+5$

f)

	$4x^2$	$-6x$
	$-10x$	$+15$

g)

	$6x^2$	$-4x$
	$+15x$	$-10$

h)

	$9x^2$	$-6x$
	$-12x$	$+8$

i)

	$8x^2$	$-10x$
	$+20x$	$-25$

**DEMO**

## Solving Quadratic Equations

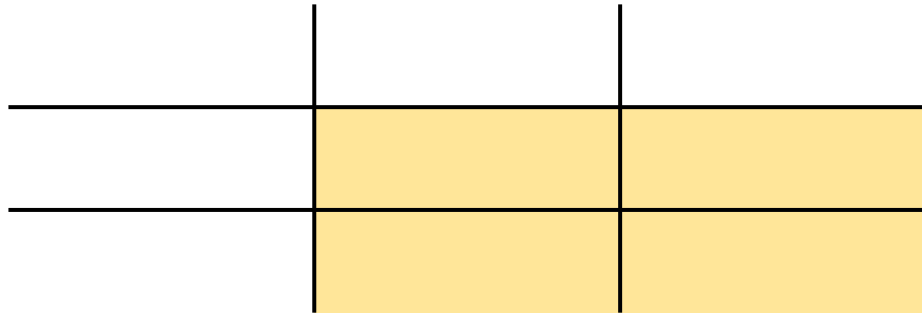
Solve:  $2x^2 + 7x + 6 = 0$

**a**            **b**            **c**

Find two numbers...

**Product = ac** = 12                      3 & 4

**Sum = b**            = 7



$$(x + 2)(2x + 3) = 0$$

$$(x + 2) = 0$$

$$x = -2$$

$$(2x + 3) = 0$$

$$2x = -3$$

$$x = -\frac{3}{2}$$



# DEMO

## Solving Quadratic Equations

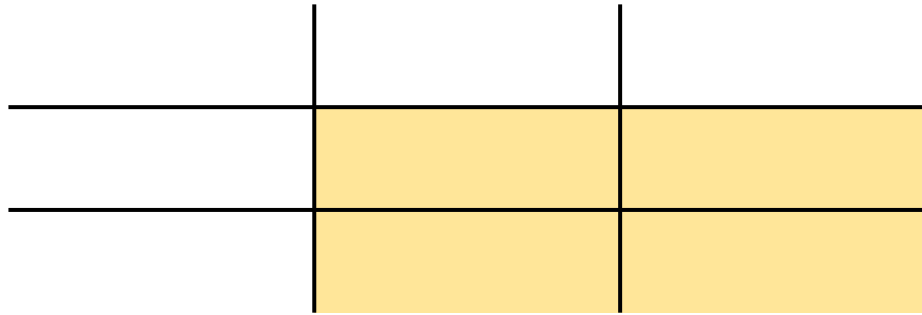
Solve:  $5x^2 + 9x + 4 = 0$

**a**            **b**            **c**

Find two numbers...

**Product = ac** = 20                    4 & 5

**Sum = b**            = 9



$$(x + 1)(5x + 4) = 0$$

$$(x + 1) = 0$$

$$x = -1$$

$$(5x + 4) = 0$$

$$5x = -4$$

$$x = -\frac{4}{5}$$

# DEMO

## Solving Quadratic Equations

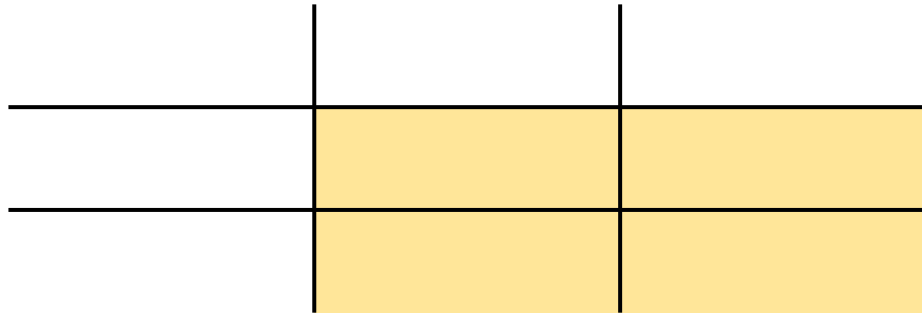
Solve:  $6x^2 + 7x - 3 = 0$

**a**            **b**            **c**

Find two numbers...

**Product = ac** = -18            -2 & 9

**Sum = b**            = 7



$$(2x + 3)(3x - 1) = 0$$

$$(2x + 3) = 0$$

$$(3x - 1) = 0$$

$$2x = -3$$

$$3x = 1$$

$$x = -\frac{3}{2}$$

$$x = \frac{1}{3}$$

# DEMO

## Solving Quadratic Equations

Solve:  $3x^2 + 8x + 1 = 0$

a            b            c

Find two numbers...

**Product = ac = 3**

**Sum = b = 8**



Try formula!


**DEMO**

## Solving Quadratic Equations

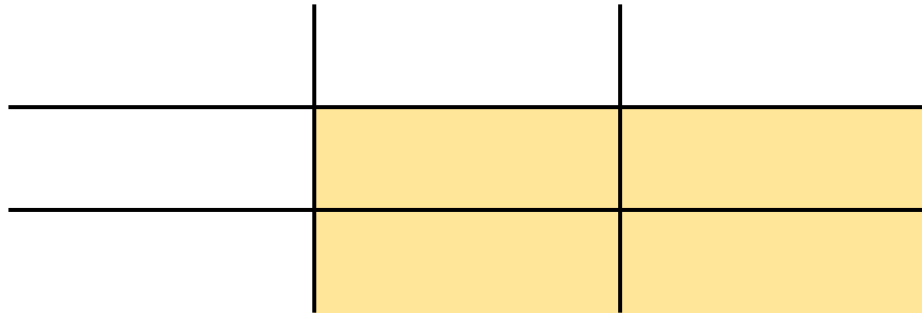
Solve:  $6x^2 - x - 1 = 0$

**a**            **b**            **c**

Find two numbers...

**Product = ac** = -6                    -3 & 2

**Sum = b**            = -1



$$(3x + 1)(2x - 1) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$3x = -1$$

$$2x = 1$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

a            b            c

Solve:  $6x^2 + 11x + 4 = 0$

a            b            c

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b** = -1

Find two numbers...

**Product = ac** = 24                      8 & 3

**Sum = b** = 11

	2x	-1
3x	6x <sup>2</sup>	-3x
+1	2x	-1


$$(3x + 1)(2x - 1) = 0$$

$$(2x + 1)(3x + 4) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$(2x + 1) = 0$$

$$(3x + 4) = 0$$

$$3x = -1$$

$$2x = 1$$

$$2x = -1$$

$$3x = -4$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

$$x = -\frac{1}{2}$$

$$x = -\frac{4}{3}$$

**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

a            b            c

Solve:  $6x^2 + 11x + 3 = 0$

a            b            c

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b** = -1

Find two numbers...

**Product = ac** = 18                      9 & 2

**Sum = b** = 11

	$2x$	$-1$
$3x$	$6x^2$	$-3x$
$+1$	$2x$	$-1$


$$(3x + 1)(2x - 1) = 0$$

$$(3x + 1)(2x + 3) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$(3x + 1) = 0$$

$$(2x + 3) = 0$$

$$3x = -1$$

$$2x = 1$$

$$3x = -1$$

$$2x = -3$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

$$x = -\frac{1}{3}$$

$$x = -\frac{3}{2}$$

**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

a            b            c

Solve:  $10x^2 - x - 2 = 0$

a            b            c

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b** = -1

	$2x$	$-1$
$3x$	$6x^2$	$-3x$
$+1$	$2x$	$-1$

$$(3x + 1)(2x - 1) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$3x = -1$$

$$2x = 1$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

Find two numbers...

**Product = ac** = -20                      -5 & 4

**Sum = b** = -1


$$(5x + 2)(2x - 1) = 0$$

$$(5x + 2) = 0$$

$$(2x - 1) = 0$$

$$5x = -2$$

$$2x = 1$$

$$x = -\frac{2}{5}$$

$$x = \frac{1}{2}$$

**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

a            b            c

Solve:  $4x^2 + 13x - 12 = 0$

a            b            c

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b** = -1

Find two numbers...

**Product = ac** = -48                      16 & -3

**Sum = b** = 13

	$2x$	$-1$
$3x$	$6x^2$	$-3x$
$+1$	$2x$	$-1$


$$(3x + 1)(2x - 1) = 0$$

$$(x + 4)(4x - 3) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$(x + 4) = 0$$

$$(4x - 3) = 0$$

$$3x = -1$$

$$2x = 1$$

$$x = -4$$

$$4x = 3$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

$$x = \frac{3}{4}$$



**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

a            b            c

Solve:  $4x^2 - 13x + 10 = 0$

a            b            c

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b** = -1

		$2x$		$-1$
$3x$		$6x^2$		$-3x$
$+1$		$2x$		$-1$

$$(3x + 1)(2x - 1) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$3x = -1$$

$$2x = 1$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

Find two numbers...

**Product = ac** = 40                      -5 & -8

**Sum = b** = -13


$$(x - 2)(4x - 5) = 0$$

$$(x - 2) = 0$$

$$(4x - 5) = 0$$

$$x = 2$$

$$4x = 5$$

$$x = \frac{5}{4}$$

**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

a            b            c

Solve:  $20x^2 - 29x + 5 = 0$

a            b            c

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b** = -1

	2x	-1
3x	6x <sup>2</sup>	-3x
+1	2x	-1

$$(3x + 1)(2x - 1) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$3x = -1$$

$$2x = 1$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

Find two numbers...

**Product = ac** = 100                      -4 & -25

**Sum = b** = -29


$$(5x - 1)(4x - 5) = 0$$

$$(5x - 1) = 0$$

$$(4x - 5) = 0$$

$$5x = 1$$

$$4x = 5$$

$$x = \frac{1}{5}$$

$$x = \frac{5}{4}$$

**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

$\begin{matrix} & a & & b & & c \end{matrix}$

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b** = -1

	$2x$	$-1$
$3x$	$6x^2$	$-3x$
$+1$	$2x$	$-1$

$$(3x + 1)(2x - 1) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$3x = -1$$

$$2x = 1$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

Solve:  $6x^2 + 11x + 4 = 0$

$$(3x + 4)(2x + 1) = 0$$

$$x = -\frac{4}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

Solve:  $10x^2 - 11x + 3 = 0$

$$(2x - 1)(5x - 3) = 0$$

$$x = \frac{1}{2} \quad \text{or} \quad x = \frac{3}{5}$$

Solve:  $6x^2 - 7x - 20 = 0$

$$(2x - 5)(3x + 4) = 0$$

$$x = \frac{5}{2} \quad \text{or} \quad x = -\frac{4}{3}$$

**DEMO**

## Solving Quadratic Equations

**YOUR TURN**

Solve:  $6x^2 - x - 1 = 0$

a
b
c

Find two numbers...

**Product = ac** = -6                      -3 & 2

**Sum = b**                      = -1

	$2x$	$-1$
$3x$	$6x^2$	$-3x$
$+1$	$2x$	$-1$

$$(3x + 1)(2x - 1) = 0$$

$$(3x + 1) = 0$$

$$(2x - 1) = 0$$

$$3x = -1$$

$$2x = 1$$

$$x = -\frac{1}{3}$$

$$x = \frac{1}{2}$$

Solve:  $15x^2 + 19x + 6 = 0$

$$(5x + 3)(3x + 2) = 0$$

$$x = -\frac{3}{5} \text{ or } x = -\frac{2}{3}$$

Solve:  $18x^2 - 21x + 5 = 0$

$$(6x - 5)(3x - 1) = 0$$

$$x = \frac{5}{6} \text{ or } x = \frac{1}{3}$$

Solve:  $14x^2 - 31x - 10 = 0$

$$(7x + 2)(2x - 5) = 0$$

$$x = -\frac{2}{7} \text{ or } x = \frac{5}{2}$$